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Period of Temporary Separation and its Effect on First Birth Interval

Introduction

THE time from marriage to first live birth is an important event in the reproductive life of a female. It has been advocated by several authors that it is relatively easier to formulate stochastic models for first birth interval (F.B.I) and estimate the fertility parameters there from Consequently, a number of papers have appeared in literature during last two decades. Potter and Parker (1964), Sheps (1964), Singh (1964, 1968) and others proposed probability models for the first conception delay. However, these and some other authors have overlooked the incidence of foetal loss before the first live birth. It is to be noted here that the F.B.I, has a direct relation with the number of incomplete conceptions occurring prior to the live birth and hence its occurrence should not be ignored while formulating a model for F. B. I.

The another remarkable fact is that the theoretical distributions for F. B. I. accounting only for biological components of fertility, often fail to describe empirical situations where the interval is highly influenced by related social customs prevalent in the society. For instance, in rural parts of India, the F. B. I, is usually lengthened due to temporary separation of married partners caused by the stay of female partner to her parent's place for some time even after marriage. This situation may arise in Western countries also following an agreement between married partners to delay the first birth voluntarily by using contraceptives or due to the presence of adolescent sterility. Thus, in the formulation of a model for F. B. I. due consideration should be given to such inoperative period.

In the present paper, a continuous time model for F. B. I. has been proposed. The model assumes the occurrence of foetal wastage prior to the first live birth. To deal with the data on first birth interval, due to reasons stated above, the period of temporary separation just after the marriage has been taken into consideration. A numerical distribution of this period has been assumed. The model has been applied to a set of data which relates to the rural parts of India.

The Model

Let T' be the time of first live birth after marriage measured in months. Obviously T' includes :

(i) the waiting time for conceptions occurring before the first live birth,
(ii) the rest periods associated with incomplete conceptions, and
(iii) the gestation period for the live birth.

As the last component is generally assumed to be a constant equal to 9 months, for convenience we derive the probability distribution of $T = T' - 9$, which is the duration of first complete conception. We make the following assumptions :

A 1 : X_0, the waiting time from marriage to first conception follows a displaced exponential distribution,

$$F_0(t) = P[X_0 \leqslant t] = 1 - e^{-\lambda(t-y)}, \; t > y, \lambda > 0 \tag{1}$$

where λ is the conception rate. Obviously, the chance of conception in the period of length y following marriage is zero, hence y may be treated as an inoperative period caused by the temporary separation of the partners.

A 2 : X_i $(i = 1, 2, \ldots)$, the time elapsed between ith and $(i+1)$th incomplete conception assumes the distribution

$$F_i(t) = P[X_i \leqslant t] = 1 - e^{-\lambda(t-h)}, \; t > h, \lambda > 0 \tag{2}$$

where h is the period of non-susceptibility following ith conception.

A 3 : N, the number of incomplete conceptions prior to the first live birth follows a geometric distribution with parameter and is given by

$$P_n = P[N = n] = (1-\theta)^n\theta, \; 0 < \theta < 1, n = 1, 2, \ldots \tag{3}$$

A 4 : Waiting times are mutually independent.

Under these assumptions the waiting time of first live birth conception, $F(t, h, y)$ is

$$F(t, y, h) = \sum_{r=0}^{[t-y/h]} \theta(1-\theta)^r \left[1 - e^{-\lambda(t-y-rh)} \sum_{s=0}^{r} \frac{\lambda^s (t-y-rh)^s}{s!}\right] \quad t > y + rh \tag{4}$$

which follows from equation (6.49), [Chapter VI of Bharucha-Reid (1960)]. $[t-y/h]$ denotes the greatest integer not exceeding $(t-y)/h$.

The model may be regarded as a conditional distribution of T for given y and h. However, Singh *et al.* (1982) have considered the variation in h assuming y to be known. Since it is difficult to have a distribution which allows for variation in all the parameters simultaneously, we assume here that y has a specific distribution, $f(y)$, among females while h is treated to be constant. The unconditional distribution of T, then, becomes

$$F(t, h) = \int_R F(t, y, h)\, f(y)\, dy, \tag{5}$$

where R denotes the range of integration.

1. *The Distribution of Y*

If a known distribution of Y is taken, the model $F(t,h)$ given by equation (5) may be obtained and it can be applied to observed data for estimating several fertility parameters. Chakrabarty (1976) assumed that the variable y takes two values viz., 0 and 12 months, with equal probability. However, it is obvious that y is a random variable taking only non-negative values. Further, a close look on the period reveals that it should have a bell-shaped distribution having a long tail on the right. In the absence of published literature, therefore, we have assumed two distributions of y.

1.1. Model M_1

Let Y has an exponential distribution given by

$$f(y) = \beta e^{-\beta y}, \qquad y > 0 \tag{6}$$

then the distribution $F(t, h)$ given in (5), becomes

$$F^*(t, h) = \sum_{r=0}^{[t/h]} \theta\,(1-\theta)^r \Big[1 - e^{-\beta(t-rh)} - \sum_{s=0}^{r} \frac{\lambda^s \beta}{(\lambda-\beta)^{s+1}}\, e^{-\beta(t-rh)}$$

$$e^{-\lambda(t-rh)} \sum_{p=0}^{s} \frac{(\lambda-\beta)^p (t-rh)^p}{p!} \tag{7}$$

1.2. Model M_2

Assuming that the period of temporary separation. y follows a chi-square distribution with n degrees of freedom (for n even), the distribution of T is given by

$$F^{**}(t,h) = \sum_{r=0}^{[t/h]} \theta(1-\theta)^r \Big[1 - e^{-\frac{1}{2}(t-rh)} \sum_{k=0}^{n/2-1} \frac{(t-rh/2)^k}{K!}$$

$$+ \sum_{s=0}^{r} \sum_{p=0}^{s} \frac{\lambda^s(-2)^p \sqrt{n/2+p}}{\sqrt{n/2}\,\sqrt{p+1}\,\sqrt{s-p+1}}$$

$$\frac{(t-rh)^{s-p}}{(1-2\lambda)\, n/2+p} \left\{ e^{-\frac{1}{2}(t-rh)} \sum_{j=0}^{n/2+p-1} \frac{(1-2)^j(t-rh/2)^j}{j!} - e^{-\lambda(t-rh)} \right\} \Big] \quad (8)$$

Mean and Variance

The simple expressions of mean and variance for both the models are easily obtained. If $\phi(s)$ be the Laplace transform of the density function of T then the mean, $E(T)$ and the variance, $V(T)$ are given as

$$E(T) = -\frac{d}{ds}\,\phi(s)\Big|_{s=0} \quad (9)$$

and

$$V(T) = E(T^2) - [E(T)]^2 \quad (10)$$

where

$$E(T^2) = \frac{d^2}{ds^2}\,\phi(s)\Big|_{s=0}$$

The mean and variance of the model M_1 are

$$E^*(T) = \frac{1+(1-\theta)\lambda h}{\lambda\theta} + \frac{1}{\beta} \quad (11)$$

$$V^*(T) = \frac{1+2\lambda h(1-\theta)+\lambda^2(1-\theta)^2h^2+\lambda^2h^2\,\theta(1-\theta)}{\lambda^2\theta^2} + \frac{1}{\beta^2} \quad (12)$$

and that of Model M_2 are

$$E^{**}(T) = \frac{1+(1-\theta)\lambda h}{\lambda\theta} + n \quad (13)$$

$$V^{**}(T) = \frac{1+\lambda^2h^2(1-\theta)+2\lambda h\,(1-\theta)}{\lambda^2\theta^2} + 2n \quad (14)$$

For any empirical distribution, one can get the estimates of any two parameters by the method of moments if other two parameters are assumed to be known.

Application of Models

It has been advocated by Potter et al (1965), Singh (1964) and Srinivasan (1967) that in India the time of first birth is largely affected by the stay of girls at their parental home for some time even after marriage. This is more frequent in rural areas where early marriage of young girls is still prevalent. For obvious reasons, therefore, the model of first birth interval which do not make an allowance for such separation periods are inappropriate. We have shown with one set of data that models M_1 and M_2 are more suitable in such situations.

To illustrate the applicability of the suggested model, the data have been taken from Chakrabarty (1976). The data relate to 'A Demographic Survey of Varanasi (Rural)' which was conducted in the year 1969-70 under the auspices of the Demographic Research Centre, Banaras Hindu University. The Survey covered about 2200 households scattered in 52 villages in Varanasi Tehsil. The information on household structure, household facilities, fertility, migration, morbidity, mortality, etc. were obtained. The details of the survey are presented in Singh et al. (1970). It is to be noted here that our data relate to the rural parts of India, hence, the presence of temporary separation may be assumed.

Table 1 shows the frequency distribution of the time of first live birth conception for females who were married at or after 16 years of age and whose marriage duration was more than 10 years on the reference date of the survey. The mean and the variance of the distribution are 2.7565 years and 6.0515 years respectively. In the light of evidences gathered in other works on the Same population, we assume 0 = 0.90 and h = 6 months.

$$\hat{\lambda} = 0.0389, \hat{\beta} = 0.2599.$$

The expected frequencies are given in the third column of Table 1.

Similarly using the equations (13) and (14), the estimates of A and n under the model M_2 are obtained as

$$\hat{\lambda} = 0.039, \hat{n} = 3.84.$$

In order to apply the Model M_1 the estimates of the parameters λ and β are obtained using equations (11) and (12). The estimates are

In the derivation of the Model M_2 we have assumed that n is an even number, while the estimated value of n is 3.84 months. Since this value is more closer to 4 months, for application purpose, we have taken n = 4 months. The ex-

TABLE 1-OBSERVED AND EXPECTED FREQUENCIES OF FIRST LIVE BIRTH CONCEPTION

Time (in months)	*Observed Frequencies*	*Expected Frequency (Using Model M^2)*	*Expected Frequency (Using Model M^2)*	*Expected Frequency (Using Model* (15))
1	*2*	*3*	*4*	5
0-15	108	119.8	118.9	140.0
15-27	97	87.6	88.3	73.9
27-39	63	58.6	58.8	51.4
39-51	44	39.1	39.2	35.8
51-63	26	25.9	28.0	25.3
63-75	16	17.2	15-4	17.4
75-87	11	11.4	11.5	14.1
87-99	7	7.6	7.6	10.0
99 and over	11	15.8	15.3	15.1
Total	383	383.0	383.0	383.0
X^2		4.719	4.187	21.91
Degrees of freedom		4	4	7

pected frequencies are presented in the fourth column of the table. As the calculated values of x^2-statistic show, both the distributions are appropriate for the observed situation.

The model for describing the time of first live birth under the similar assumptions as given above, except the period of temporary separation y after marriage, is given by

$$F(t, h) = \sum_{r=0}^{[t/h]} \theta(1-\theta)^r \left[1 - e^{-\lambda(t-rh)} \sum_{s=0}^{r} \frac{\{\lambda(t-rh)\}^s}{s!}\right], t > rh. \qquad (15)$$

In order to apply it to the same data, the estimate of the parameter A is obtained as 0.0342 using the expression of mean. The expected frequencies under the column 5 relates to the distribution (15). The calculated value of X^2-s tic reveals that the model fails to describe the empirical data.

Conclusion

Although the proposed models have been derived under the strong assumptions of time-homogeneity and constancy of the risk parameter A over a heterogeneous group of females, the models are simple for use and describe the variations in the observed data. In so far as the expected frequencies are concerned, both the distributions seem to be identical. Since the average of a X^2-distribution with *n* degrees of freedom is *n,* it is clear that the average length of the period of temporary separation in the surveyed population is about 4 months. This is also evident from the value of (3 in the Model *MI* (the average length here is 1/p = 4 months approximately). These values are also in agreement with the estimated value of *y* (4.2 months) in Singh *et al.* (1982). The assumed average length of *y* (6 months) taken by Chakrabarty (1976), thus, seems incorrect.

It can be now concluded that the period of temporary separation caused due to social customs or otherwise plays an important role in the first birth interval and hence it should be considered in the related probability models. Models giving allowance for such inoperative period may also be helpful in assessing the changes in fertility due to the adoption of some family planning methods of hundred percent effectiveness just after the marriage by married couples in order to delay the first birth.

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